MAT247-Statistical Applications

FORMULA SHEET

Sample Mean

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum xf}{\sum f}$$

Sample Variance

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 f - \frac{\left(\sum xf\right)^2}{n}\right)$$

Sample Standard Deviation

$$s = \sqrt{sample\ \text{var}\, iance} = \sqrt{s^2}$$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cap B) = 0$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Compliment Law

$$P(\bar{B}) = 1 - P(B)$$

Discrete Random Variables

$$E(X) = \mu = \sum xP(x)$$
$$Var(X) = \sum x^2 P(x) - (\mu)^2$$

Binomial Distribution

If the random variable X has a binomial such that $X \sim Bin(n, p)$, then

$$P(X = x) = ({}^nC_x)(p)^x(1-p)^{n-x}$$
$$mean = np$$
$$Variance = np(1-p)$$

Poisson Distribution

If the random variable X has a Poisson distribution, then

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$mean = \lambda$$
$$Variance = \lambda$$

Normal Distribution

If the random variable X has a normal distribution with mean μ and variance σ^2, i.e. $X \sim N(\mu, \sigma)$, then

$$Z = \frac{X - \mu}{\sigma}$$

$100(1-\alpha)\%$ **Confidence intervals for** μ

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$$ (normal population; σ known)

$100(1-\alpha)\%$ **Confidence intervals for** μ

$$\bar{x} \pm z\frac{s}{\sqrt{n}}$$ (large sample; σ unknown)

$100(1-\alpha)\%$ **Confidence intervals for** μ

$$\bar{x} \pm t\frac{s}{\sqrt{n}}$$; (small sample; σ unknown)

$100(1-\alpha)\%$ **Confidence intervals for** p

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Hypothesis testing for μ

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$ (large sample; σ unknown)

Hypothesis testing for μ

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}};\quad df = n-1$$(small sample; σ unknown)

Hypothesis testing for the proportion

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

Chi-Squared Test of Independence

Expected value

$$E = \frac{Row\ total \times Column\ Total}{n}$$

Test Statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Regression Analysis

- $\hat{y} = a + bx$
- $SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$
- $SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$
- $\boldsymbol{SS_{xy} = \sum xy - \frac{\sum x \sum y}{n}}$
- $b = \frac{SS_{xy}}{SS_{xx}}$
- $a = \bar{y} - b\bar{x}$ or $a = \frac{\sum y}{n} - b\frac{\sum x}{n}$
- coefficient of correlation**:**

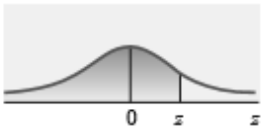
  $$r = \frac{SS_{xy}}{\sqrt{SS_{xx} \times SS_{yy}}}$$

- coefficient of determination : $r^2 = b\frac{SS_{xy}}{SS_{yy}}$

THE POSTIVE STANDARD NORMAL TABLE

Table IV Standard Normal Distribution Table (continued from previous page)

The entries in this table give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or positive.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

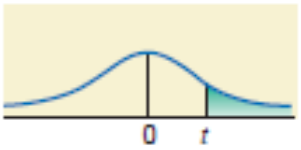
THE NEGATIVE STANDARD NORMAL TABLE

The entries in this table give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or negative.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table V The t Distribution Table

The entries in this table give the critical values of t for the specified number of degrees of freedom and areas in the right tail.



	Area in the Right Tail under the t Distribution Curve					
df	.10	.05	.025	.01	.005	.001
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
31	1.309	1.696	2.040	2.453	2.744	3.375
32	1.309	1.694	2.037	2.449	2.738	3.365
33	1.308	1.692	2.035	2.445	2.733	3.356
34	1.307	1.691	2.032	2.441	2.728	3.348
35	1.306	1.690	2.030	2.438	2.724	3.340

Table V The *t* Distribution Table (continued

	Area in the Right Tail under the		
df	.10	.05	.025
36	1.306	1.688	2.028
37	1.305	1.687	2.026
38	1.304	1.686	2.024
39	1.304	1.685	2.023
40	1.303	1.684	2.021
41	1.303	1.683	2.020
42	1.302	1.682	2.018
43	1.302	1.681	2.017
44	1.301	1.680	2.015
45	1.301	1.679	2.014
46	1.300	1.679	2.013
47	1.300	1.678	2.012
48	1.299	1.677	2.011
49	1.299	1.677	2.010
50	1.299	1.676	2.009
51	1.298	1.675	2.008
52	1.298	1.675	2.007
53	1.298	1.674	2.006
54	1.297	1.674	2.005
55	1.297	1.673	2.004
56	1.297	1.673	2.003
57	1.297	1.672	2.002
58	1.296	1.672	2.002
59	1.296	1.671	2.001
60	1.296	1.671	2.000
61	1.296	1.670	2.000
62	1.295	1.670	1.999
63	1.295	1.669	1.998
64	1.295	1.669	1.998
65	1.295	1.669	1.997
66	1.295	1.668	1.997
67	1.294	1.668	1.996
68	1.294	1.668	1.995
69	1.294	1.667	1.995
70	1.294	1.667	1.994
71	1.294	1.667	1.994
72	1.293	1.666	1.993
73	1.293	1.666	1.993
74	1.293	1.666	1.993
75	1.293	1.665	1.992
∞	1.282	1.645	1.960

Table VI Chi-Square Distribution Table

The entries in this table give
number of degre

	Area in the Right Tail				
df	.995	.990	.975	.950	
1	0.000	0.000	0.001	0.004	
2	0.010	0.020	0.051	0.103	
3	0.072	0.115	0.216	0.352	
4	0.207	0.297	0.484	0.711	
5	0.412	0.554	0.831	1.145	
6	0.676	0.872	1.237	1.635	
7	0.989	1.239	1.690	2.167	
8	1.344	1.646	2.180	2.733	
9	1.735	2.088	2.700	3.325	
10	2.156	2.558	3.247	3.940	
11	2.603	3.053	3.816	4.575	
12	3.074	3.571	4.404	5.226	
13	3.565	4.107	5.009	5.892	
14	4.075	4.660	5.629	6.571	
15	4.601	5.229	6.262	7.261	
16	5.142	5.812	6.908	7.962	
17	5.697	6.408	7.564	8.672	
18	6.265	7.015	8.231	9.390	
19	6.844	7.633	8.907	10.117	
20	7.434	8.260	9.591	10.851	
21	8.034	8.897	10.283	11.591	
22	8.643	9.542	10.982	12.338	
23	9.260	10.196	11.689	13.091	
24	9.886	10.856	12.401	13.848	
25	10.520	11.524	13.120	14.611	
26	11.160	12.198	13.844	15.379	
27	11.808	12.879	14.573	16.151	
28	12.461	13.565	15.308	16.928	
29	13.121	14.256	16.047	17.708	
30	13.787	14.953	16.791	18.493	
40	20.707	22.164	24.433	26.509	
50	27.991	29.707	32.357	34.764	
60	35.534	37.485	40.482	43.188	
70	43.275	45.442	48.758	51.739	
80	51.172	53.540	57.153	60.391	
90	59.196	61.754	65.647	69.126	
100	67.328	70.065	74.222	77.929	